Scientific contribution/Review

The Big Bang Theory

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Abstract:
In this contribution, the Big Bang theory is presented to describe the evolution of our universe. The Big Bang theory states that the universe started in a point with an infinite energy density about 14 billion years ago and has been expanding ever since. Crucial for this theory is the Hubble's discovery that observed galaxies are moving away from us. The evolution of our universe has mostly been governed by the forces of gravitation; therefore, Newton's law of gravity is used to derive the equations that describe how the size of our universe is changing with time. From these equations, it is possible to estimate the age of our universe. In this contribution we present the model of the universe and derivation of the estimation of the universe age.

Keywords: The Big Bang; Universe; Gravity; Hubble's constant; Age of the universe

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1. Introduction

The Big Bang theory describes the evolution of the universe. The essence of the theory is that the universe started in a point with an infinite energy density and has been expanding ever since (Peebles et al., 1994). Consequently, the galaxies today are moving away from each other, which can be observed by measuring the amount of red shift in the light emitted by a galaxy. We will use Hubble's discovery that the galaxies, which are further away from us, are actually moving away from us faster. With the aid of the Newton's law of gravitation, we will derive the equations which govern the evolution of the universe and estimate the age of the universe.

2. Methods

Edwin Powell Hubble discovered that almost all galaxies in the universe are moving away from us. This was a revolutionary discovery since theories before that assumed a static universe. Furthermore, by measuring the red shift in the light emitted by galaxies, he discovered an even more important fact: the galaxies that are further away from us are moving away from us faster (Osterbrock et al., 1993). Hubble's law states that the velocity with which a certain galaxy is moving away from us is proportional to the distance to that galaxy (**Figure 1**). Note that some neighbouring galaxies could be approaching each other because of gravitational forces.

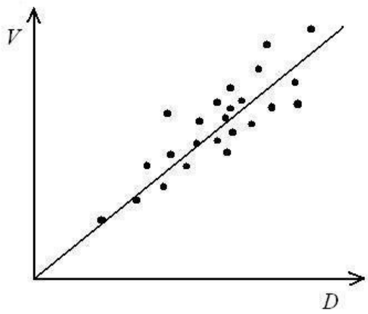

Figure 1. The velocity (V) of the galaxies that are moving away from us as a function of the distance (D) of those galaxies relative to us (Liddle, 2003).

We can visualise space as an elastic band which is expanding. If both ends of the elastic band are moving away from each other with a constant velocity then any two points on the elastic band are also moving away from each other. Larger the distance between the two points on the elastic, higher the relative velocity between them. The observer can be placed on any point on the elastic band and they will see all the other points moving away from them. We use a simple 1D model, where galaxies are homogeneously distributed along the x-axis (**Figure 2**). Coordinate x is in this formulation used to number the galaxies. The distance between the neighbouring galaxies is denoted by the scale factor $a(t)$, which is a function of time because the universe in not static.

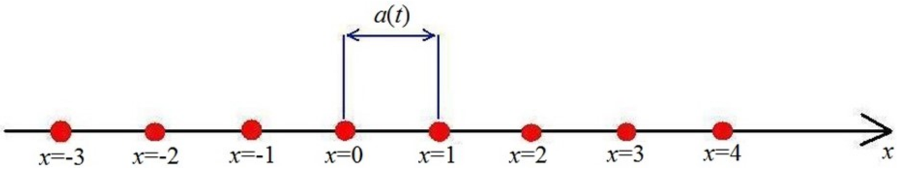

Figure 2. Galaxies that are moving away from each other are homogeneously distributed along the x-axis.

The distance between any two galaxies in these coordinates can be written as:

$$D = a(t)\Delta x. \tag{1}$$

The relative velocity between any two galaxies is therefore:

$$V = \dot{a}(t)\Delta x = \frac{\dot{a}(t)}{a(t)}a(t)\Delta x = \frac{\dot{a}(t)}{a(t)}D, \tag{2}$$

where the dot denotes the derivative with respect to time. Hubble's constant is introduced as:

$$H = \frac{\dot{a}(t)}{a(t)}\ . \tag{3}$$

Hubble's constant is a constant in space, which means that it is the same throughout the universe at a certain time. Nevertheless, it is changing with time when the universe is expanding because the scale factor $a(t)$ changing with time. By inserting Eq. (3) into Eq. (2) we obtain:

$$V = HD. \tag{4}$$

The velocity of a certain galaxy moving away from us is therefore proportional the distance to that galaxy, which is also an experimental fact presented in **Figure 1**, where Hubble's constant H is the slope of the line presented in the graph.

To study the evolution of the universe, we will use Newton's gravitational law. Newton's cosmology assumes that gravitation is the only force that governs the universe. This is actually not correct because today the evolution of the Universe in mostly dictated by the dark energy, while right after the Big Bang, radiation was the most important (Liddle, 2003). Nevertheless, classic gravitation is sufficient to describe the most of the universe's evolution so far. We will assume that universe is homogeneous and isotropic. We will write equations relative to the virtual origin in $x = 0$ as presented in **Figure 3**.

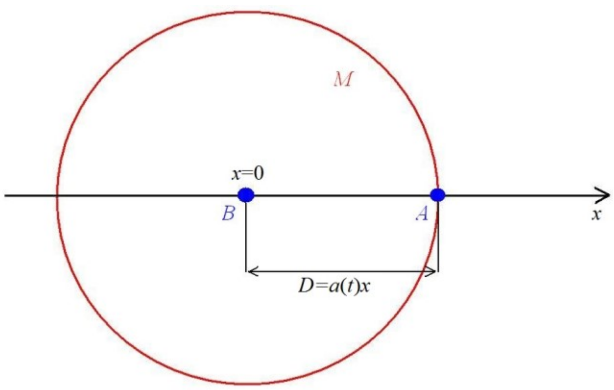

Figure 3. Coordinates that are used to derive the time dependence of the scale factor $a(t)$. Red circle represents a spherical shell with the radius D, which contains homogeneously distributed galaxies. M is the mass of all galaxies inside the shell.

First, we write the energy of the galaxy located at the point A as this galaxy is moving away from the origin at the point B (**Figure 3**). From the Newton's shell theorem, we know that the mass inside the shell with the radius D is acting on the galaxy located at the point A as it would all be concentrated in the centre of the shell at the point B. Furthermore, all the mass outside of the shell has no gravitational effect on the galaxy located at the point A. Total energy of the galaxy at the point A, which is moving away from the origin, is written as a sum of kinetic and potential energy:

$$\frac{1}{2}mV^2 - \frac{MmG}{D} = E, \tag{5}$$

where m is the mass of that galaxy and V its velocity. M is the mass of all galaxies inside the shell and G is gravitational constant. Total energy of the galaxy E is conserved with time. By inserting Eqs. (1) and (2) into Eq. (5) we get:

$$x^2\dot{a}(t)^2 - \frac{2MG}{xa(t)} = \frac{2E}{m} \ . \tag{6}$$

Note that Δx from Eqs. (1) and (2) was replaced with x since the distance D is measured from the origin as presented in **Figure 3**. Mass M can be written as a product of the mass density ρ of the galaxies inside the shell and the volume of the spherical shell with the radius D: $V_K = 4\pi D^3/3 = 4\pi x^3 a(t)^3/3$. Eq. (6) is thus transformed into:

$$x^2\dot{a}(t)^2 - \frac{8G\rho\pi x^2 a(t)^2}{3} = \frac{2E}{m} \ , \tag{7}$$

where the mass density ρ is changing when the universe is expanding. Lefthand side of Eq. (7) is proportional to x^2. The equation can be solved if we assume that also the righthand side is proportional to x^2, which is reasonable because the energy of the galaxies further away should be larger since they are moving away from us faster. We define constant k:

$$k = -\frac{2E}{mx^2} \ . \tag{8}$$

Energy has to be proportional to x^2 for k to be constant. With this assumption, the result does not depend on the chosen pair of galaxies because all terms with x^2 cancel out. If equation (8) is considered in Eq. (7), we get the Friedmann equation:

$$H^2 = \frac{\dot{a}(t)^2}{a(t)^2} = \frac{8}{3}G\rho\pi - \frac{k}{a(t)^2} \ , \tag{9}$$

which tells us how $a(t)$ is changing with time. Solutions of Eq. (9) are presented in the next section.

6. Results

Solutions of equation (9) depend on the value of the constant k. There are three qualitatively different solutions. If $k > 0$, the absolute value of gravitational energy is larger than kinetic energy. In this case, gravity stops the expansion and the universe starts to shrink back after some time (**Figure 4**, blue line). When $k < 0$, the kinetic energy is larger compared to the absolute value of gravitational energy and the expansion of the universe never stops (**Figure 4**, green line). And lastly, for $k = 0$, the absolute value of gravitational energy is the same as kinetic energy. In this case, the velocity of expansion is always decreasing and the expansion stops in infinity (**Figure 4**, red line). Observations show that our universe is close to the critical case $k = 0$ (Liddle, 2003). To solve the differential Eq. (9), we write the mass density as:

$$\rho = \frac{M}{a(t)^3}, \tag{10}$$

where the mass M inside the volume element $a(t)^3$ is constant. If Eq. (10) is considered in Eq. (9), we obtain the following differential equation for $a(t)$:

$$\frac{\dot{a}(t)^2}{a(t)^2} = \frac{8GM\pi}{3a(t)^3} - \frac{k}{a(t)^2}. \tag{11}$$

Solutions of Eq. (11) for different values of k are presented in **Figure 4**.

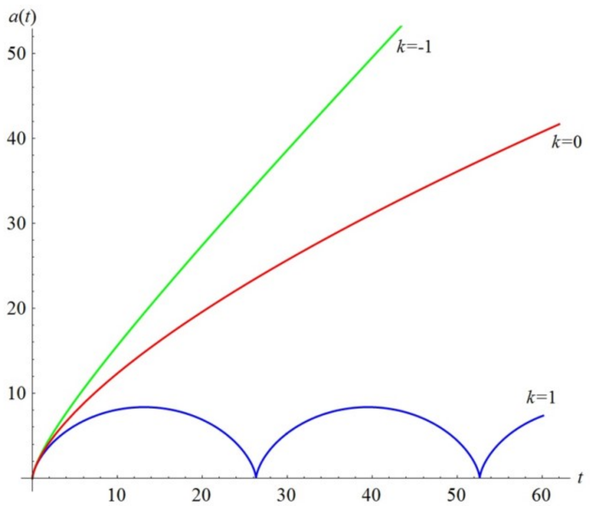

Figure 4. Time dependence of the scale factor $a(t)$ obtained by solving the differential Eq. (11) for different values of the constant k. Blue line represents the evolution of the closed universe ($k > 0$). Red line represents the evolution of the critical universe ($k = 0$). Green line represents the evolution of the open universe ($k < 0$). Other parameters in Eq. (11): $G = 1$, $M = 1$.

To estimate the age of our universe, we will write the solution of the differential Eq. (11) for $k = 0$. The equation can be solved by assuming the solution:

$$a(t) = Ct^p, \tag{12}$$

where C and p are constants, which are determined by inserting Eq. (12) into Eq. (11). We obtain the following solution:

$$a(t) = \sqrt[3]{6\pi MG}\, t^{2/3} \propto t^{2/3}. \tag{13}$$

At the time $t = 0$ the scale factor $a(t) = 0$ and the whole universe is concentrated in a single point. This is the time of the Big Bang and from that point the universe starts to expand and has been expanding ever since. Therefore, time t is measured from the Big

Bang. This solution is presented as a red line in Figure 4. If we plug the solution from Eq. (13) into the Eq. (3), we get the time dependence of the Hubble's constant for our universe:

$$H = \frac{2}{3t} \propto \frac{1}{t} \,. \tag{14}$$

By measuring the Hubble's constant, we can estimate the age of our universe:

$$t_V = \frac{2}{3H} \,. \tag{15}$$

The age of our universe is approximately 14 billion years (Hawley et al., 2005). Hubble's constant can be measured by measuring the distance to different galaxies and their velocity relative to us. In **Figure 1**, Hubble's constant *H* is the slope of the line presented in the graph.

7. Discussion

This contribution presents the description of the universe within the Newton's law of gravitation, which assumes that gravitation is the only force that governs the evolution of the universe. Today, the expansion of the universe is actually dictated by the dark energy, while in the early stages the radiation played the most important role (Liddle, 2003). Nevertheless, the main part of the evolution of our universe was governed by gravitational forces (Liddle, 2003). We derived the equation, which tells us how the size of the universe has been changing with time since the Big Bang, which allowed us to estimate the age of our universe. For more accurate and comprehensive description, we would have to use the Einstein's theory of general relativity. Our universe is close to the critical universe, which means that the absolute value of gravitational energy has a similar value as kinetic energy. Considering only this fact, the velocity of the expansion of our universe should decrease over time. Nevertheless, the accelerated expansion has been observed. This is a consequence of the dark energy effect, which causes the exponential expansion of the universe. We are now in the dark energy dominated era and our universe will expand even faster in the future (Hawley et al. 2005).

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Conflicts of Interest: The authors declare no conflict of interest.

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